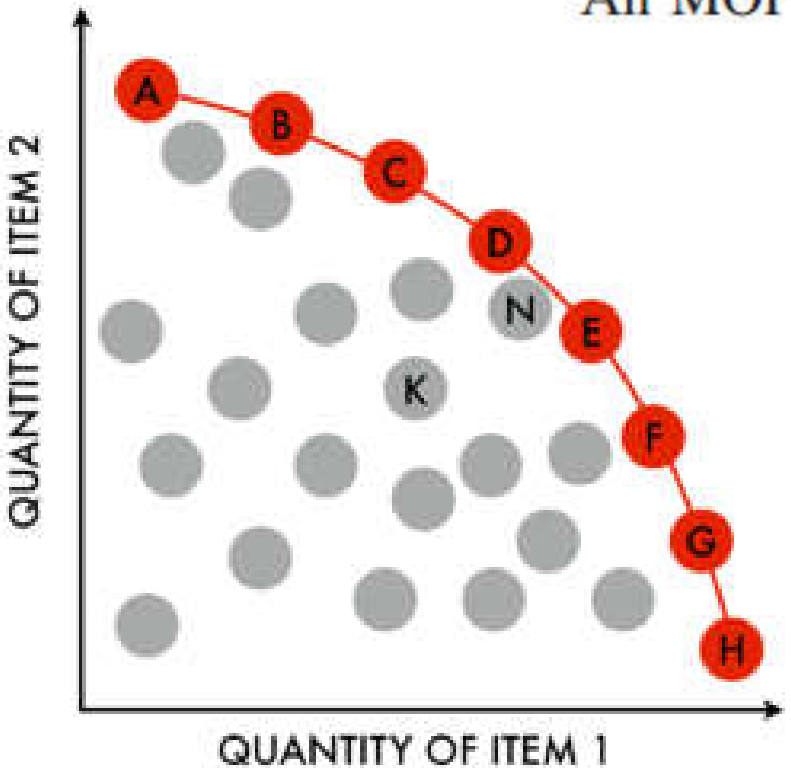
# MOEA/D
# with sorting-and-selection

2016.12.02

张谋钊

# 多目标优化问题



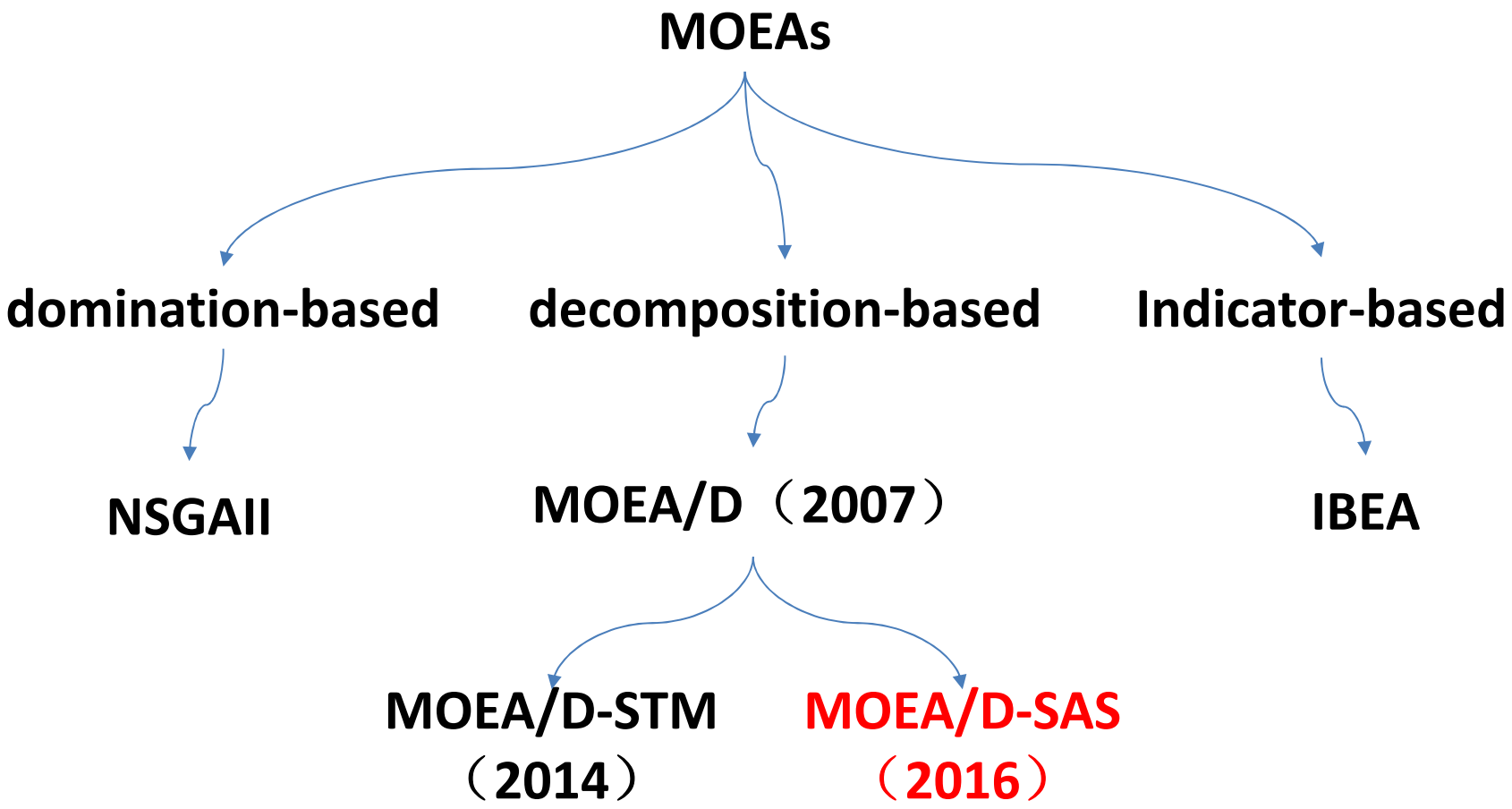

An MOP can be defined as follows:

$$\text{minimize} \quad F(x) = (f_1(x), \ldots, f_m(x))^T$$
$$\text{subject to} \quad x \in \Omega$$

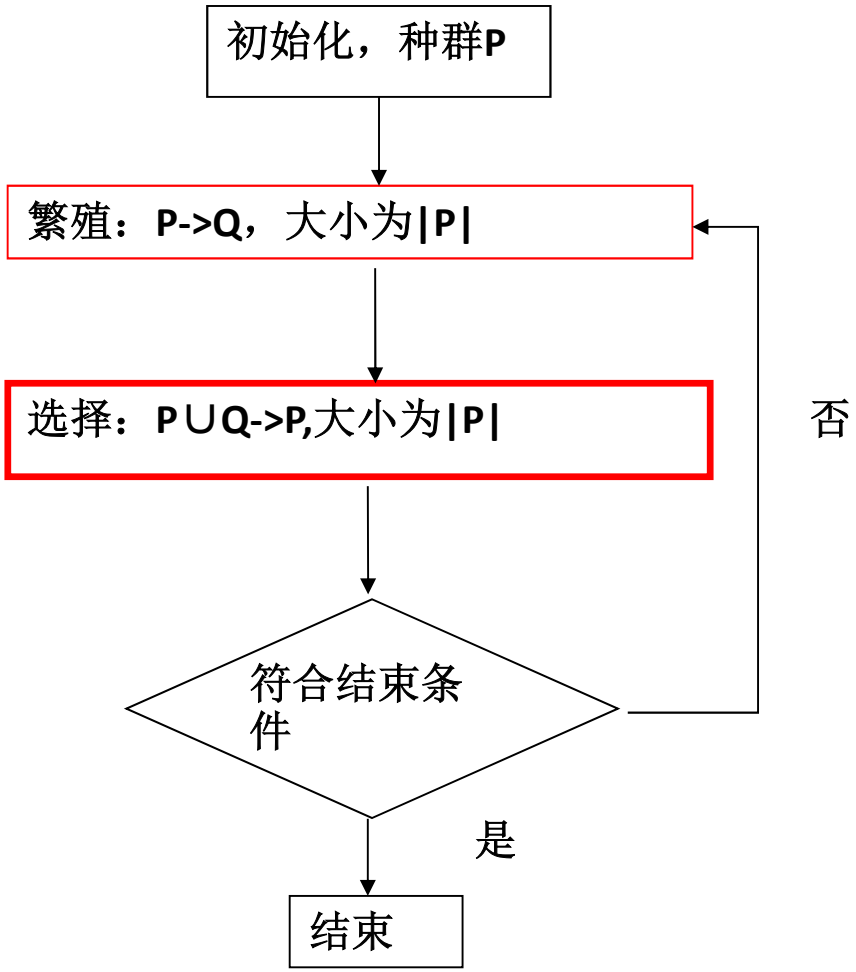
**Pareto front(PF)**

- Convergence
- Diversity(Distribution)

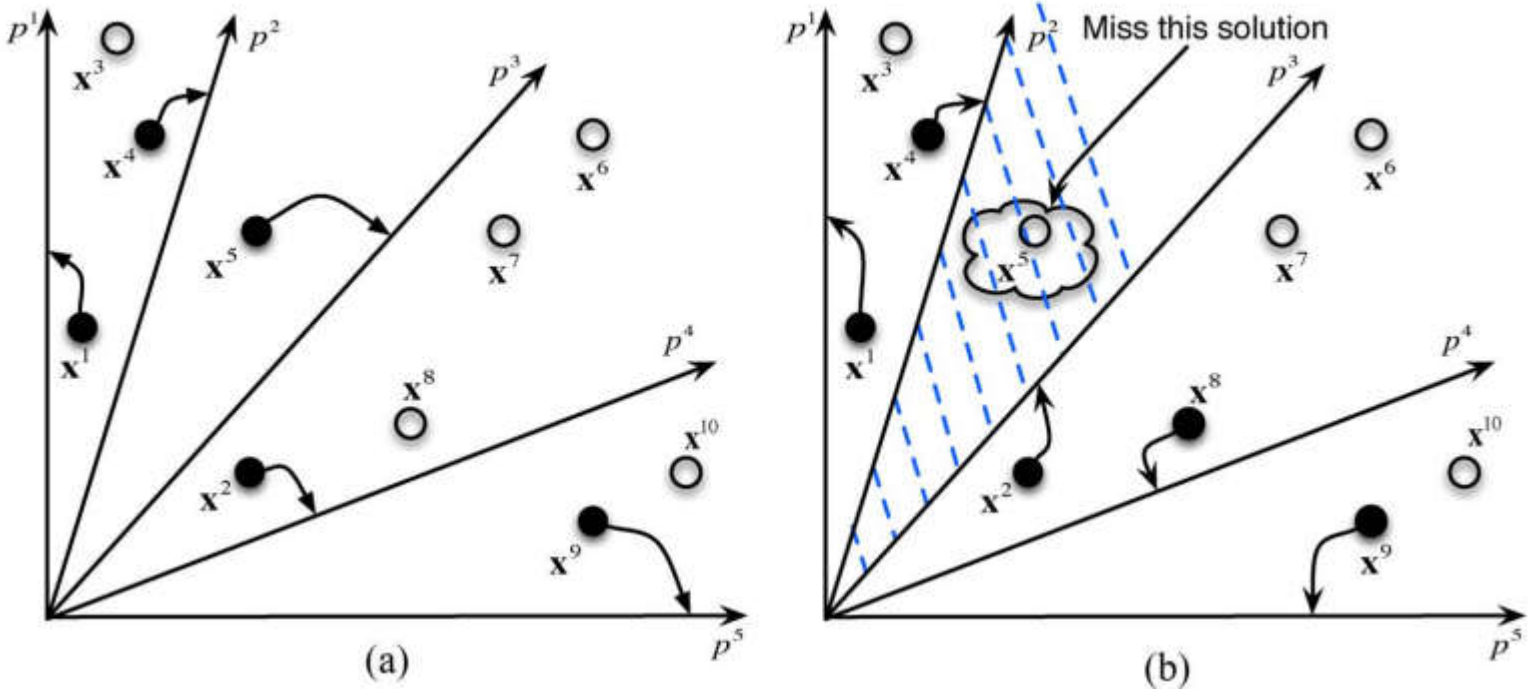
# 解决方案


MOEAs
domination-based
decomposition-based
Indicator-based
NSGAII
MOEA/D（2007）
IBEA
MOEA/D-STM
（2014）
MOEA/D-SAS
（2016）

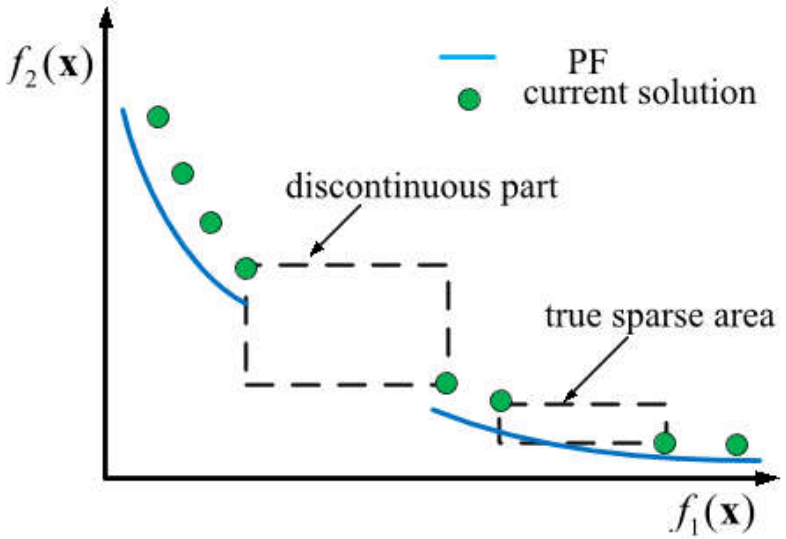
# 进化迭代过程



# MOEA/D-STM vs. MOEA/D



# 存在问题




1.问题PF不规则时，多样性差

- 多样性受权重向量的先验准确性影响

2.MOEA/D-STM选择算法复杂度高

3.已有多样性增强算法(niche-counting)为粗粒度

# MOEA/D-SAS

- SAS(Sorting And Selection)—选择算子
  - DBS(Decomposition-Based-Sorting)

    对每个子问题，计算最靠近该问题的L个解的子问题目标值，并排序
  - ABS(Angle-Based-Selection)

    从DBS得到的集合中选择解，大部分选择收敛性好的解，小部分选择分布性好的解

- 优点：
  - 一个子问题可对应多个或零个解
  - DBS只排序离子问题近的部分解，复杂度低
  - ABS加入具体细粒度提高多样性的解

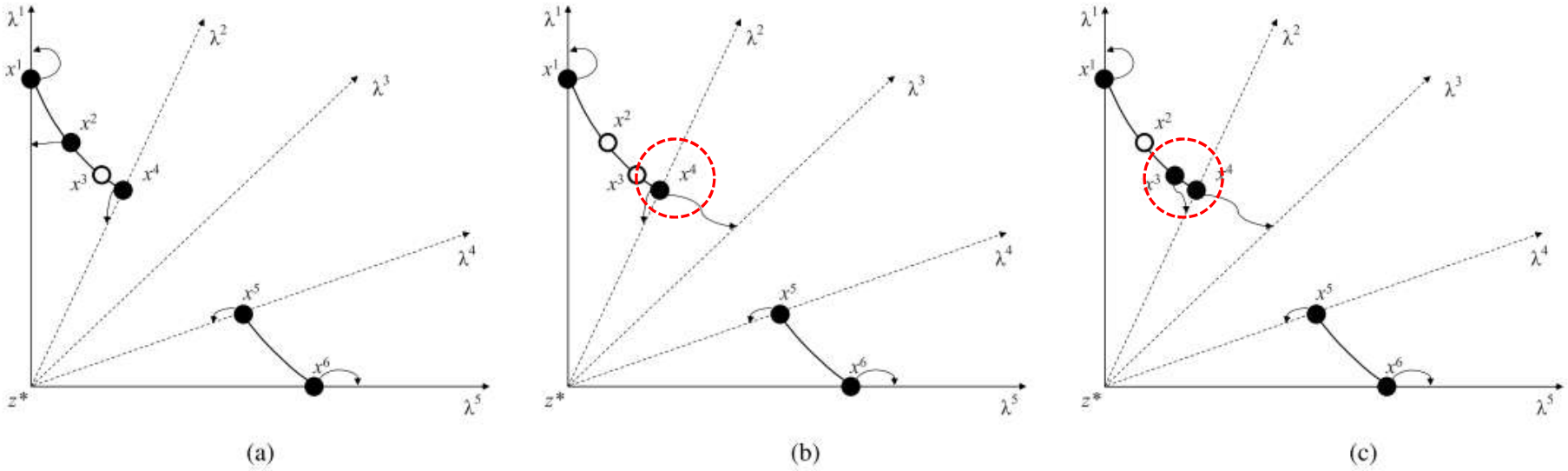



Fig. 1. Illustrative examples of different selection results. (a) Ideal selection result. Selection result obtained by (b) MOEA/D and (c) MOEA/D-STM.

MOEA/D-SAS MOEA/D MOEA/D-STM

**Algorithm 2:** DBS($Z, z^*, L$)

**Input**:

1) $Z$: the solution set;
2) $z^*$: the ideal objective vector;
3) $L$: the number of closest solutions to each subproblem.

**Output**:

1) $L$ solution sets: $Q^1, \ldots, Q^L$;

```
Q^1 = Q^2 = ... = Q^L = ∅.
for j = 1 to N do
    P^j(L) = Associate(Z, λ^j);        /* Associate subproblem j with the L closest solutions P^j in Z, based on (5). */
    for k = 1 to L do
        g(P^j(k)|λ^j, z*);             /* Computing the aggregation function values. */
    end
    P^j = Sort(P^j);  /* Sort P^j in an ascending order, based on g(P^j|λ^j, z*) values. */
end
for k = 1 to L do
    for j = 1 to N do
        Q^k = Q^k ⋃ {P^j(k)};
    end
end
return Q^1, ..., Q^L ;
```

$$\text{angle}(x, \lambda^j) = \arccos\left(\frac{(F(x) - z^*)^T \lambda^j}{\|F(x) - z^*\| \|\lambda^j\|}\right). \tag{5}$$

**Algorithm 1:** SAS($Z, z^*, L, N$)

**Input**:

1) $Z$: the solution set;
2) $z^*$: the ideal objective vector;
3) $L$: the number of closest solutions to each subproblem;
4) $N$: the size of $P$.

**Output**:

1) the population $P$;
2) the number of selected fronts $\alpha$.

**Step 1 Sorting:**

$P = \emptyset$;
$i = 1$;
**do**
  $[Q^{(i-1)*L+1}, \ldots, Q^{i*L}]$ = DBS $(Z, z^*, L)$;
  **for** $k = (i-1) * L + 1$ **to** $i * L$ **do**
    $P = P \bigcup Q^k$;
    $Z = Z \setminus Q^k$;
  **end**
  $i = i + 1$;
**while** $|P| < N$;

**Step 2 Selection:**

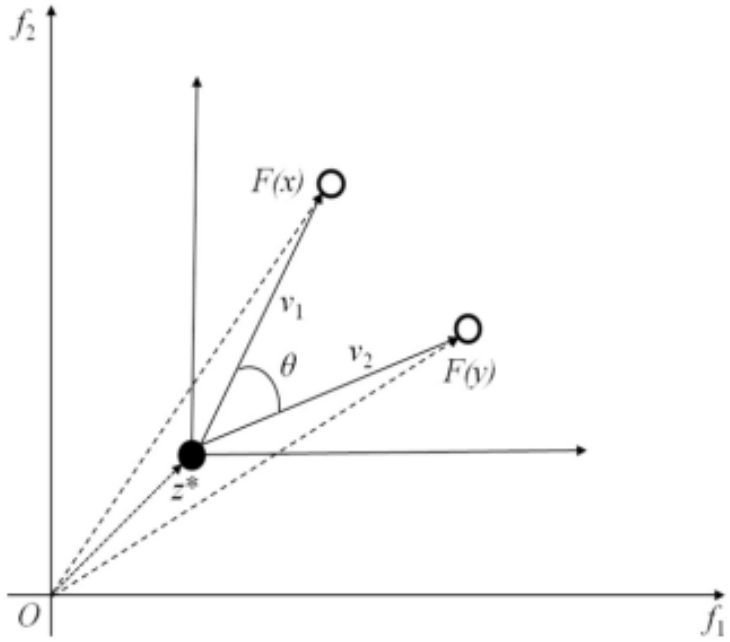
$P = \emptyset$;
$k = 1$;
**while** $|P \bigcup Q^k| \leq N$ **do**
  $P = P \bigcup Q^k$;
  $k = k + 1$;
**end**
**if** $|P \bigcup Q^k| > N$ **then**
  $A = Q^k \setminus P$;
  $P$ = ABS$(A, P, z^*, N)$;
  $\alpha = k$;
**else**
  $\alpha = k - 1$;
**end**
**return** $P$ and $\alpha$;

**Algorithm 3:** ABS($A$, $P$, $z^*$, $N$)

**Input**: two populations $A$ and $P$, the ideal objective vector $z^*$, size of $P$: $N$

**Output**: the population $P$

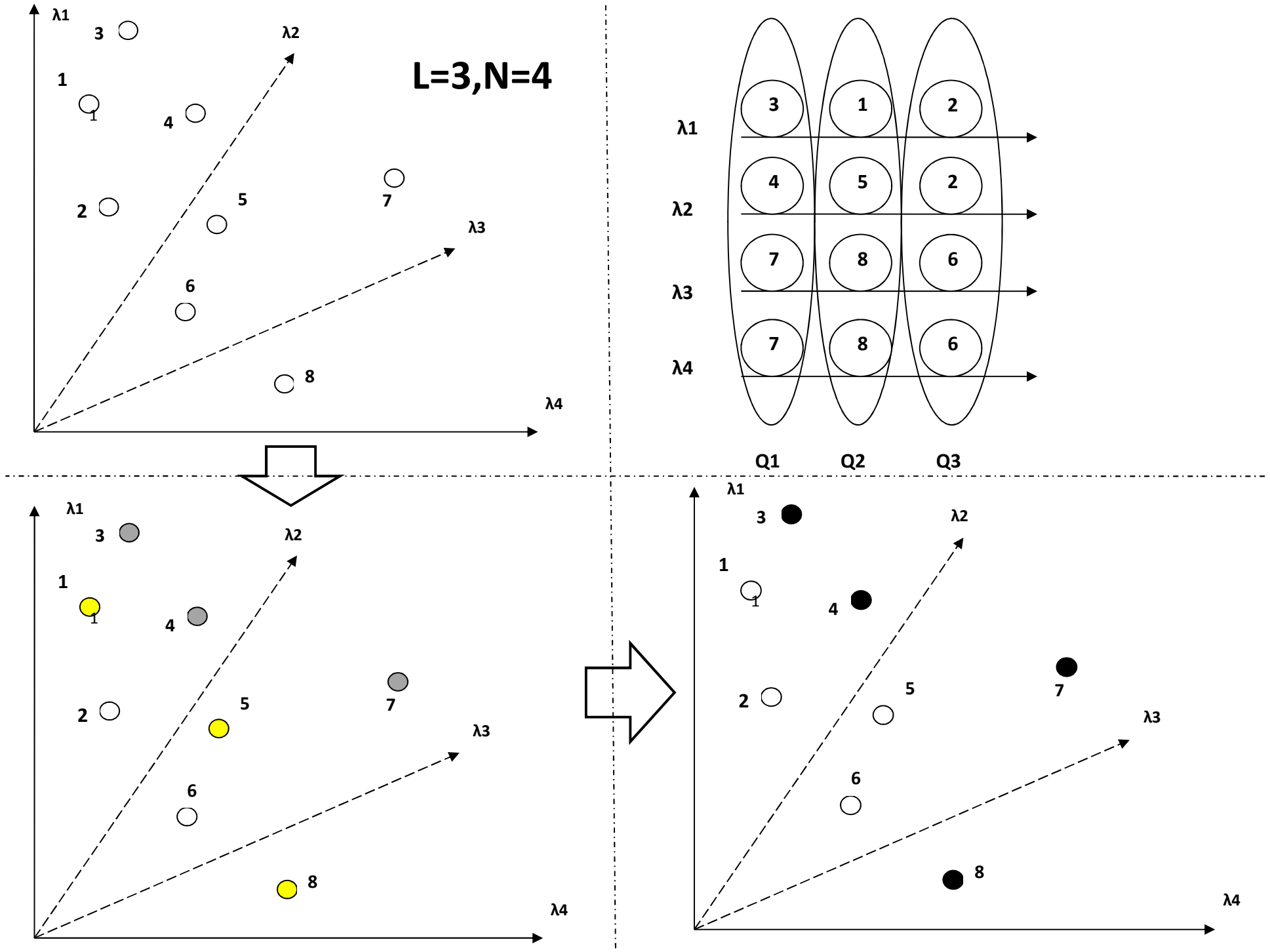
```
foreach x^i ∈ A do
    θ_i ← min_{x^j ∈ P}{angle(x^i, x^j)};
end
for i ← 1 to |A| do
    k ← argmax_k{θ_k};
    if |P| < N then
        A ← A \ {x^k};
        θ ← θ \ {θ_k};
        P ← P ⋃ {x^k};
        foreach x^j ∈ A do
            θ_j ← min{θ_j, angle(x^k, x^j)};
        end
    else
        break;
    end
end
return P;
```




$$\text{angle}(x, y) = \arccos\left(\frac{v_1^T v_2}{\|v_1\|\|v_2\|}\right) \tag{6}$$

where

$$v_1 = F(x) - z^*$$
$$v_2 = F(y) - z^*.$$


L=3,N=4
λ1
λ2
λ3
λ4
1
2
3
4
5
6
7
8
Q1
Q2
Q3

**Algorithm 4:** MOEA/D-SAS

**Input:**

1) MOP(1);
2) a stopping criterion;
3) $N$: the number of subproblems; the population size of $P$ and $Y$;
4) $\boldsymbol{\lambda}^1, \ldots, \boldsymbol{\lambda}^N$: a set of $N$ weight vectors;
5) $T$: the size of the neighborhood for each subproblem.

**Output:** population $P$.

**Step 1 Initialization:**

**a)** Compute the Euclidean distances between any two weight vectors and obtain $T$ closest weight vectors to each weight vector. For each $i = 1, \ldots, N$, set $B(i) = \{i_1, \ldots, i_T\}$ where $\lambda^{i_1}, \ldots, \lambda^{i_T}$ are the $T$ closest weight vectors to $\lambda^i$.

**b)** Generate an initial population $P = \{x^1, \ldots, x^N\}$ randomly.

**c)** Initialize $z^* = (z_1^*, \ldots, z_m^*)^T$ by setting $z_i^* = \min\{f_i(x^1), \ldots, f_i(x^N)\}$.

**d)** Initialize $\alpha = 2 * N$.

**Step 2 New Solution Generation:**

For each $i = 1, \ldots, |P|$, do:

**a) Selection of the Mating Solutions:**

1) Associate each solution $x^i$ with its closest subproblem $k$ based on (5).
2) If $rand(0, 1) < \delta$, then set $D$ to the set of solutions associated with all the subproblems in $B(k)$, else, set $D = P$.

**b) Reproduction:** Set $x^{r_1} = x^i$ and randomly select two indices $r_2$ and $r_3$ from $D$, and then generate a new solution $y^i$ from $x^{r_1}$, $x^{r_2}$ and $x^{r_3}$ by DE.

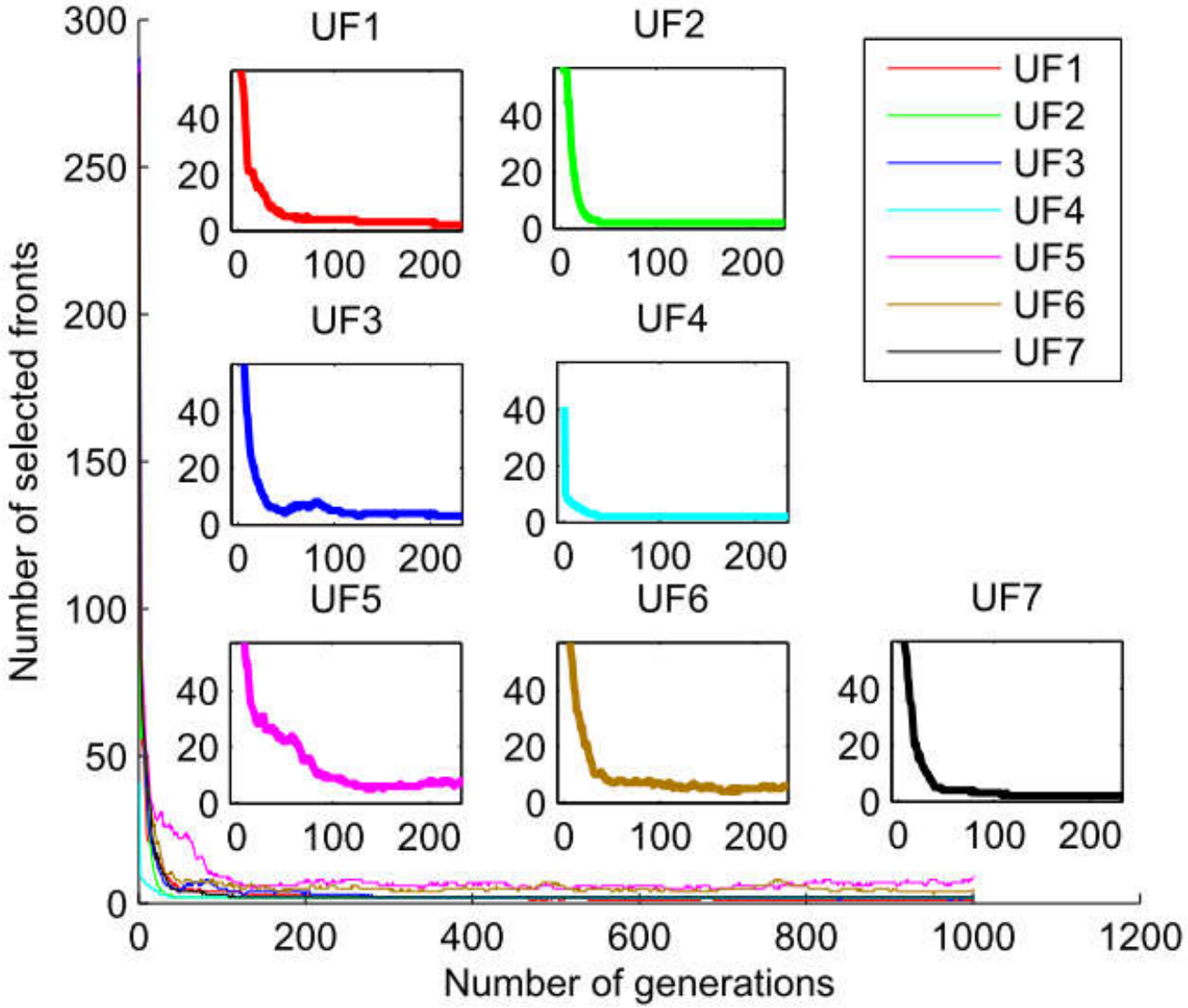
**c) Evaluation** $y^i$ **:** $FV^i = F(y^i)$.

**d) Update of** $z^*$ **:** For each $j = 1, \ldots, m$, if $z_j^* > f_j(y^i)$, then set $z_j^* = f_j(y^i)$.

**Step 3 Sorting-and-Selection:** Set $L = \min\{\alpha + T, 2N\}$, $[P, \alpha] = \mathrm{SAS}(P \bigcup Y, z^*, L, N)$.

**Step 4 Stopping Criteria:** If stopping criteria is satisfied, then stop and output $P$. Otherwise, go to **Step 2**.

# 关于α


UF1
UF2
UF3
UF4
UF5
UF6
UF7
Number of selected fronts
Number of generations
0
50
100
150
200
250
300
0
200
400
600
800
1000
1200
0
20
40
0
100
200

# 实验

*Test Problems*

- **UF**

  Contains ten unconstrained MOP test instances (UF1–UF10) from the CEC2009 MOEA competition.Seven of them (UF1–UF7) are 2-objective test functions, and the rest (UF8–UF10) are 3-objective functions

- **DTLZ**

  All DTLZ instances can be scaled to any number of objectives and decision variables

- *Performance Metrics*

$$\mathrm{IGD}(P, P^*) = \frac{1}{|P^*|} \sum_{v \in P^*} \mathrm{dist}(v, P) \qquad (11)$$

# *MOEA/D-SAS VS. Classical MOEAs*

TABLE III
MEAN AND STANDARD DEVIATION VALUES OF IGD, OBTAINED BY MOEA/D-SAS, MOEA/D, MOEA/D-DE, MSOPS-II, AND NSGA-II ON UF AND DTLZ INSTANCES

| instance | *IGD* | | | | |
|---|---|---|---|---|---|
| | MOEA/D-SAS | MOEA/D | MOEA/D-DE | MSOPS-II | NSGA-II |
| UF1 | **1.644E-03 (5.52E-05)** | 9.807E-02 (5.92E-02) † | 2.439E-03 (4.94E-04) † | 7.425E-02 (5.44E-03) † | 8.390E-02 (1.17E-02) † |
| UF2 | **7.162E-03 (2.57E-03)** | 7.712E-02 (3.40E-02) † | 1.118E-02 (3.21E-03) † | 5.716E-02 (2.10E-02) † | 3.272E-02 (2.32E-03) † |
| UF3 | **9.987E-03 (8.25E-03)** | 2.912E-01 (5.23E-02) † | 2.539E-02 (2.12E-02) † | 3.141E-01 (1.75E-02) † | 7.031E-02 (1.14E-02) † |
| UF4 | 5.699E-02 (2.52E-03) | 9.186E-02 (6.82E-03) † | 6.767E-02 (2.80E-03) † | **5.671E-02 (4.09E-03)** | 7.606E-02 (1.35E-02) † |
| UF5 | **2.358E-01 (3.14E-02)** | 1.178E+00 (1.62E-01) † | 2.901E-01 (4.56E-02) † | 3.437E-01 (9.80E-02) † | 6.793E-01 (9.88E-02) † |
| UF6 | **9.963E-02 (5.76E-02)** | 4.958E-01 (1.58E-01) † | 1.868E-01 (1.34E-01) † | 2.985E-01 (2.29E-01) † | 3.217E-01 (7.60E-02) † |
| UF7 | **2.480E-03 (6.28E-04)** | 2.546E-01 (2.50E-01) † | 4.067E-03 (9.31E-04) † | 4.181E-02 (7.21E-03) † | 3.504E-01 (8.65E-03) † |
| UF8 | **3.356E-02 (4.03E-03)** | 7.324E-02 (2.35E-02) † | 6.213E-02 (7.45E-03) † | 1.912E-01 (5.38E-03) † | 2.671E-01 (5.44E-02) † |
| UF9 | **2.925E-02 (3.88E-03)** | 1.228E-01 (4.90E-02) † | 6.111E-02 (3.85E-02) † | 2.354E-01 (3.42E-02) † | 1.840E-01 (6.91E-02) † |
| UF10 | 5.236E-01 (6.87E-02) | 4.827E-01 (7.29E-02) | 4.971E-01 (4.44E-02) | **2.490E-01 (1.20E-01)** † | 6.632E-01 (6.81E-02) † |
| DTLZ1 | **1.081E-02 (1.21E-04)** | 1.600E-02 (5.88E-05) † | 1.611E-02 (1.32E-04) † | 1.573E+00 (6.27E-01) † | 4.158E+00 (2.56E+00) † |
| DTLZ2 | **2.897E-02 (3.68E-05)** | 3.727E-02 (8.30E-05) † | 3.725E-02 (8.69E-05) † | 3.757E-02 (2.16E-03) † | 5.972E-02 (7.90E-03) † |
| DTLZ3 | **3.363E-02 (3.33E-04)** | 3.868E-02 (2.52E-04) † | 3.885E-02 (1.86E-04) † | 4.866E+00 (2.64E+00) † | 1.458E+01 (8.01E+00) † |
| DTLZ4 | 3.404E-02 (2.41E-04) | 1.292E-01 (1.83E-01) † | 5.971E-02 (7.34E-02) † | **3.321E-02 (4.57E-04)** | 7.318E-02 (2.84E-02) † |
| DTLZ5 | 3.598E-03 (1.43E-04) | 4.394E-03 (1.11E-05) † | 4.402E-03 (9.04E-06) † | 1.578E-02 (3.82E-03) † | **1.836E-03 (1.34E-04)** † |
| DTLZ6 | **3.685E-03 (1.95E-04)** | 4.420E-03 (3.18E-06) † | 4.583E-03 (1.56E-04) † | 9.139E-01 (7.54E-02) † | 5.388E-03 (1.53E-03) † |
| DTLZ7 | **6.837E-02 (5.13E-02)** | 3.586E-01 (2.82E-01) † | 1.151E-01 (2.70E-03) † | 1.240E-01 (1.29E-01) † | 7.828E-01 (1.20E-01) † |

# *MOEA/D-SAS VS. MOEA/D-STM*

TABLE IV
MEAN AND STANDARD DEVIATION VALUES OF IGD, OBTAINED BY MOEA/D-SAS AND MOEA/D-STM ON UF AND DTLZ INSTANCES

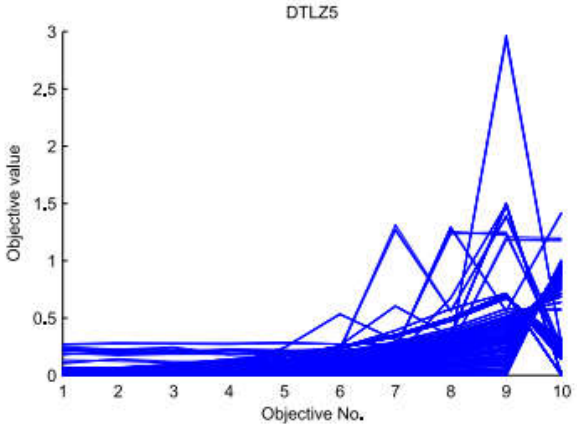
| instance | *IGD* | | |
|---|---|---|---|
| | MOEA/D-SAS | MOEA/D-STM | p-value |
| UF1 | **1.644E-03 (5.52E-05)** | 1.980E-03 (6.32E-05) | 3.02E-11 |
| UF2 | 7.162E-03 (2.57E-03) | 7.074E-03 (1.84E-03) | 0.7845 |
| UF3 | 9.987E-03 (8.25E-03) | **3.721E-03 (2.48E-03)** | 5.09E-06 |
| UF4 | 5.699E-02 (2.52E-03) | 5.614E-02 (3.75E-03) | 0.2340 |
| UF5 | **2.358E-01 (3.14E-02)** | 2.522E-01 (2.45E-02) | 2.15E-02 |
| UF6 | 9.963E-02 (5.76E-02) | 8.202E-02 (3.75E-02) | 0.6520 |
| UF7 | **2.480E-03 (6.28E-04)** | 2.658E-03 (6.35E-04) | 4.86E-03 |
| UF8 | **3.356E-02 (4.03E-03)** | 6.331E-02 (1.04E-02) | 4.50E-11 |
| UF9 | 2.925E-02 (3.88E-03) | 2.748E-02 (2.26E-03) | 0.0555 |
| UF10 | **5.236E-01 (6.87E-02)** | 1.213E+00 (2.09E-01) | 3.02E-11 |
| DTLZ1 | **1.081E-02 (1.21E-04)** | 2.367E+00 (7.91E+00) | 3.82E-10 |
| DTLZ2 | **2.897E-02 (3.68E-05)** | 3.000E-02 (2.11E-04) | 3.02E-11 |
| DTLZ3 | **3.363E-02 (3.33E-04)** | 2.499E+00 (7.85E+00) | 6.07E-11 |
| DTLZ4 | **3.404E-02 (2.41E-04)** | 8.748E-02 (1.10E-01) | 3.02E-11 |
| DTLZ5 | **3.598E-03 (1.43E-04)** | 7.083E-03 (2.63E-05) | 3.02E-11 |
| DTLZ6 | **3.685E-03 (1.95E-04)** | 7.091E-03 (8.72E-07) | 3.02E-11 |
| DTLZ7 | **6.837E-02 (5.13E-02)** | 1.627E-01 (4.52E-02) | 5.07E-10 |

# *MOEA/D-SAS VS. NSGAIII*

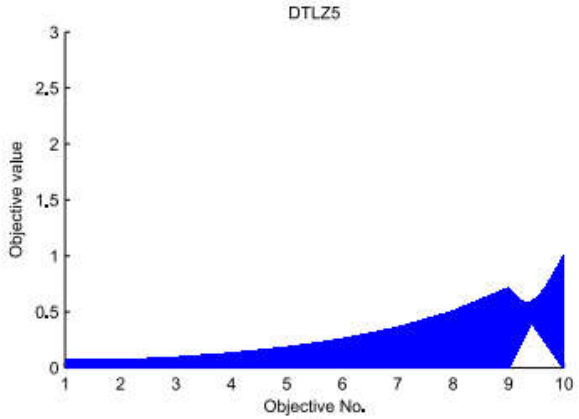
TABLE VII
MEAN AND STANDARD DEVIATION VALUES OF IGD, OBTAINED BY MOEA/D-SAS AND NSGA-III ON DTLZ INSTANCES

| M | instance | MOEA/D-SAS | NSGA-III | p-value |
|---|---|---|---|---|
| 5 | DTLZ1 | 6.667E-02 (1.447E-03) | **5.193E-02 (1.753E-04)** | 3.02E-11 |
| | DTLZ2 | 2.337E-01 (9.387E-03) | **1.332E-01 (2.645E-04)** | 3.02E-11 |
| | DTLZ3 | 1.851E+00 (2.254E+00) | **1.341E-01 (3.200E-03)** | 3.02E-11 |
| | DTLZ4 | 2.108E-01 (1.228E-02) | **1.331E-01 (1.266E-04)** | 3.02E-11 |
| | DTLZ5 | **2.043E-02 (1.241E-03)** | 9.550E-02 (2.769E-02) | 3.02E-11 |
| | DTLZ6 | **2.029E-02 (1.010E-03)** | 1.364E+00 (9.325E-02) | 3.02E-11 |
| | DTLZ7 | 5.926E-01 (4.230E-02) | **4.116E-01 (1.255E-01)** | 6.52E-09 |
| 8 | DTLZ1 | **9.579E-02 (7.327E-04)** | 2.001E-01 (5.059E-02) | 1.11E-06 |
| | DTLZ2 | 4.999E-01 (1.240E-02) | 5.535E-01 (2.323E-01) | 0.1858 |
| | DTLZ3 | 5.735E-01 (6.574E-01) | 6.850E-01 (2.437E-01) | 0.2282 |
| | DTLZ4 | 5.010E-01 (6.964E-03) | **3.654E-01 (1.242E-03)** | 3.02E-11 |
| | DTLZ5 | **2.524E-02 (8.936E-04)** | 3.256E-01 (1.126E-01) | 3.02E-11 |
| | DTLZ6 | **2.391E-01 (5.993E-01)** | 4.014E+00 (4.411E-01) | 3.02E-11 |
| | DTLZ7 | **1.364E+00 (1.642E-01)** | 2.286E+00 (3.537E-01) | 3.02E-11 |
| 10 | DTLZ1 | **9.380E-02 (7.515E-04)** | 2.318E-01 (4.330E-02) | 8.48E-09 |
| | DTLZ2 | **5.215E-01 (5.599E-03)** | 7.467E-01 (2.207E-01) | 7.96E-03 |
| | DTLZ3 | 1.267E+00 (1.907E+00) | **8.784E-01 (9.647E-02)** | 4.94E-05 |
| | DTLZ4 | 5.675E-01 (5.178E-03) | **4.159E-01 (2.819E-03)** | 3.02E-11 |
| | DTLZ5 | **2.064E-02 (2.453E-04)** | 4.465E-01 (1.092E-01) | 3.02E-11 |
| | DTLZ6 | **1.789E-01 (4.577E-01)** | 4.961E+00 (4.647E-01) | 3.02E-11 |
| | DTLZ7 | **1.765E+00 (9.144E-02)** | 4.572E+00 (7.510E-01) | 3.02E-11 |
| 15 | DTLZ1 | **7.082E-02 (5.327E-03)** | 3.121E-01 (8.653E-03) | 3.02E-11 |
| | DTLZ2 | **4.510E-01 (3.219E-02)** | 1.014E+00 (2.716E-02) | 3.02E-11 |
| | DTLZ3 | **3.593E-01 (2.267E-01)** | 1.137E+00 (3.675E-01) | 5.57E-10 |
| | DTLZ4 | **1.478E-01 (1.124E-02)** | 1.031E+00 (2.401E-02) | 6.77E-05 |

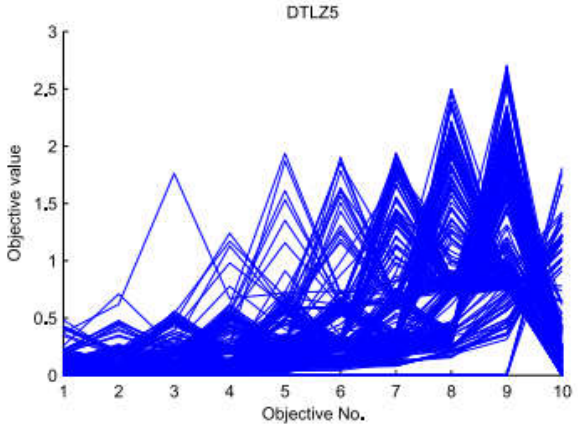
# *MOEA/D-SAS VS. NSGAIII*



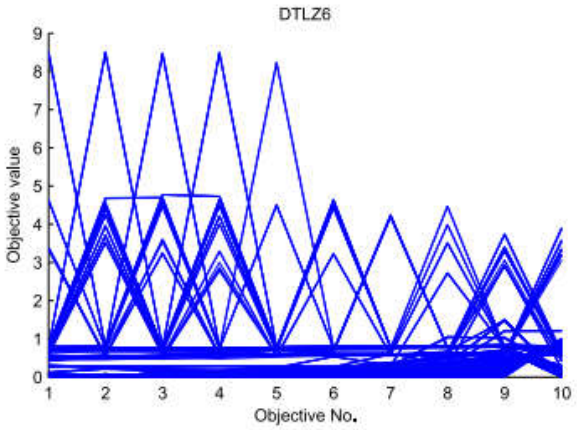

(g)

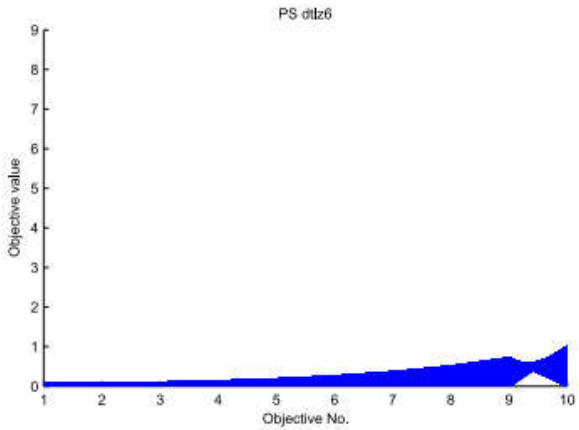


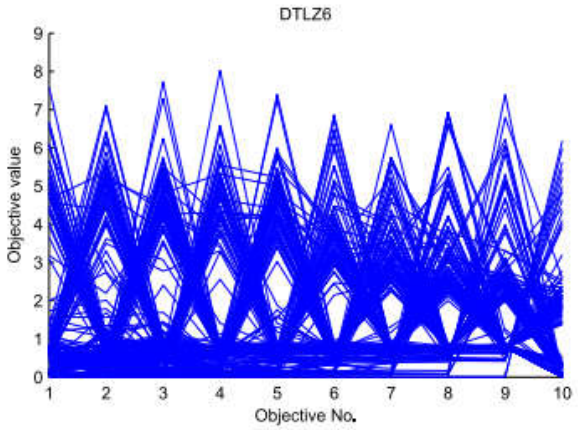

(h)




(i)



***Thank you for listening***