Many-Objective Evolutionary Algorithm: Objective Space Reduction and Diversity Improvement

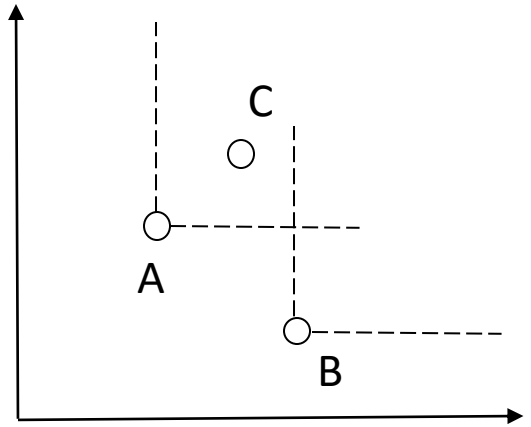
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问题描述：

解决多目标问题的一个重要方法是非支配排序，即

最终，由A、B等非支配解组成
pareto 前沿



普通的基于非支配排序的遗传算法在解决高维目标优化问题（MaOPs）时所面临的问题主要有：

1、选择压力低下导致的大量的解都是非支配的
2、维数灾难使求解的目标空间太大，效率低下

这篇文章针对上述两个问题，提出了一种高效的方法，主要由以下两个步骤组成：
1、Objective space reduction——search for target points——overcome the ineffectiveness of the pareto dominance relation

2.Diversity improvement——preserve diversity and update target points

Stage 1 :Objective space reduction

- 思想：当我们将高维的空间约束到低维空间，我们就能找到真实pareto area的某些边界点，通过这些点大概能推测出pareto area的边界。这些点应该具有特征1、包含某一维的极值2、位置在边界上的角落。得到这些边界点后，我们就可以进行第二阶段
- 边界点——target point
- 方法——ASF

ASF

- Achievement Scalarizing Function (ASF)
- 大部分求超平面的主要方法，如NSGAIII
- 目的：确定种群中的边界点

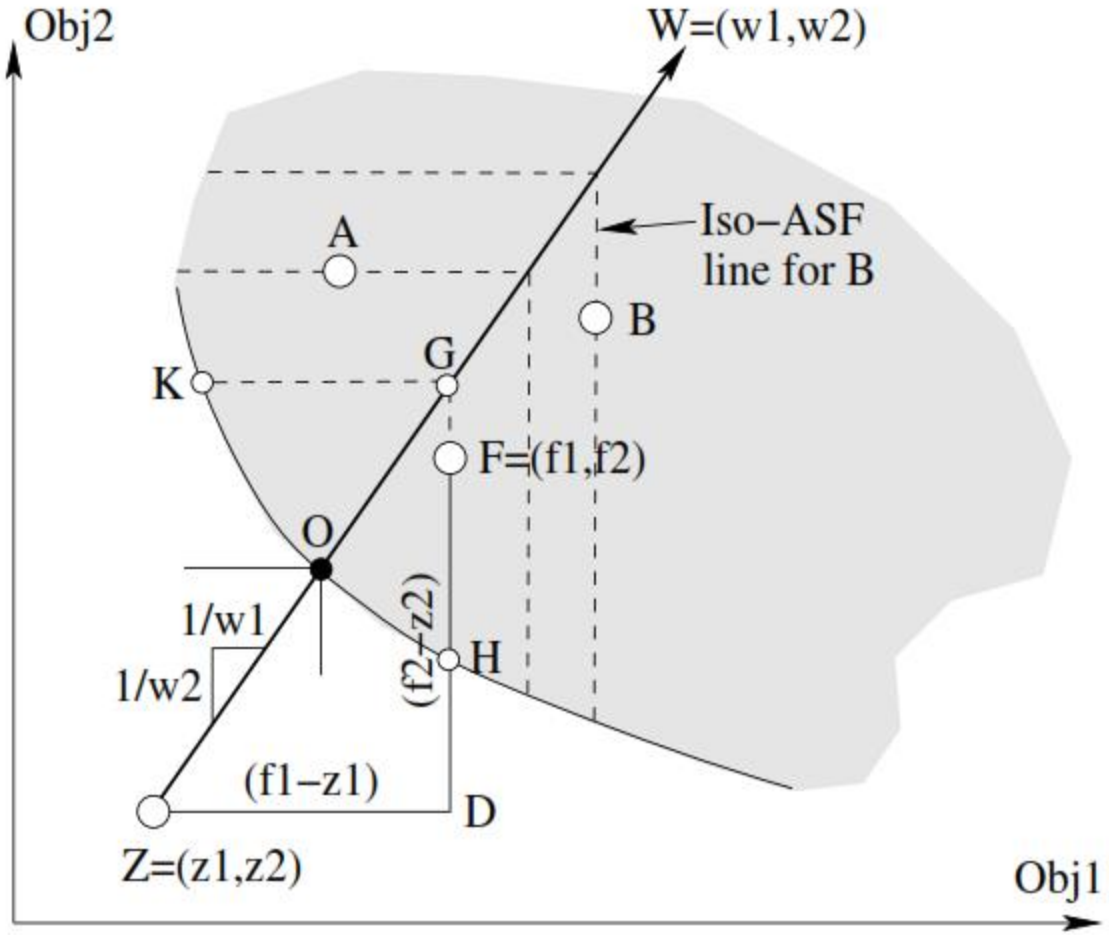
$$\mathrm{ASF}\left(j,\lambda^{i}\right)=\max_{k=1:M}\left\{\left(f_{j,k}-z_{k}^{\mathrm{ideal}}\right)\Big/\lambda_{k}^{i}\right\}\qquad(1)$$

z_k^{ideal}是理想点，fj,k是第j个个体在第k维的函数值，则$\left(f_{j,k}-z_k^{\mathrm{ideal}}\right)$是归一化过程

$$\lambda^{i}=\left(10^{-6},\ldots,1,10^{-6},\ldots,10^{-6}\right)^{T}\qquad(2)$$

是第i维的权重向量

$$\mathrm{ASF}\left(j, \lambda^i\right) = \max_{k=1:M} \left\{ \left(f_{j,k} - z_k^{\mathrm{ideal}} \right) \Big/ \lambda_k^i \right\} \qquad (1)$$



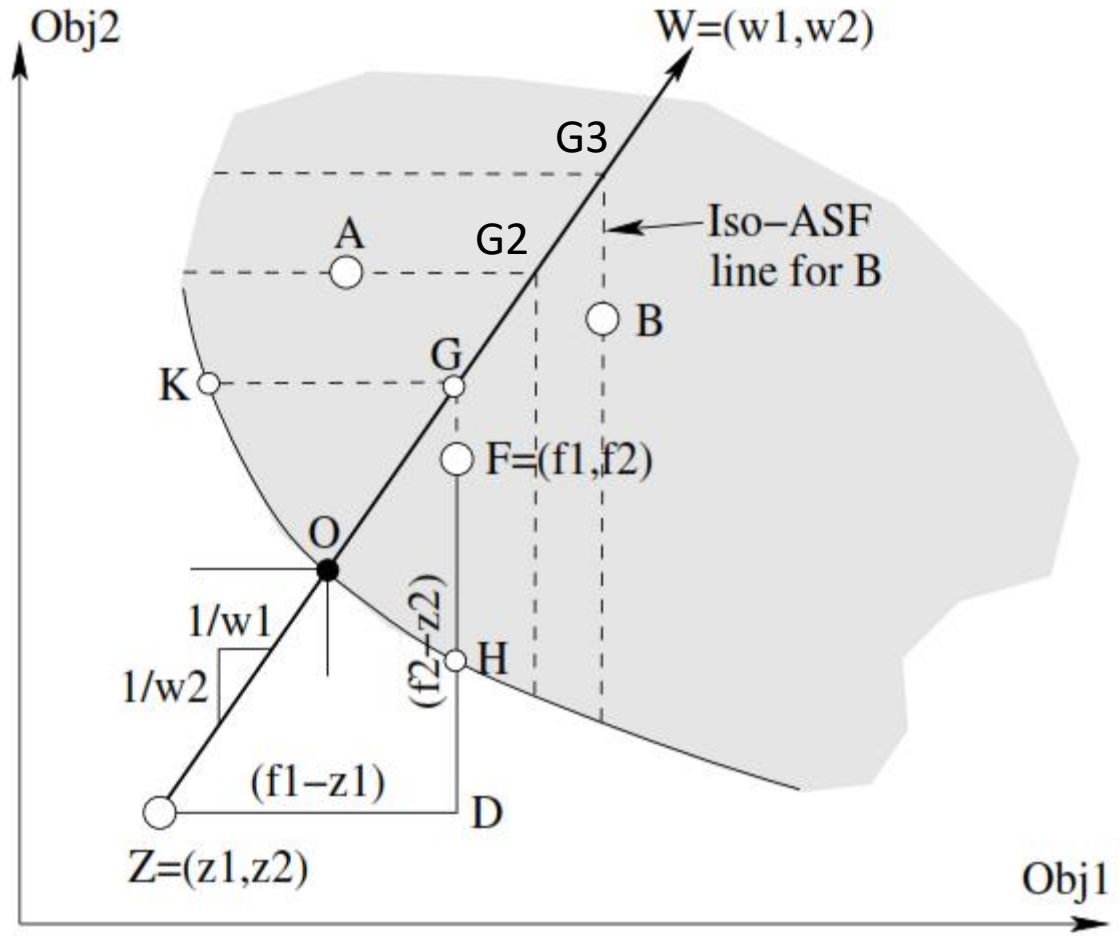
设 w=(1,1)
考虑ZW向量上的一点G。可以发现
ASF(G，w)=MAX{f1-z1/1,f2-z2/1}
考虑GD上的点F
F(纵)<G(纵)=G(横)=F(横)
GD上的点得ASF值均为（f1-z1）
GK上的点得ASF值均为（f2-z2）

当w改变时，如（1,2）
点G的(f1-z1)/w1=(f2-z2)/w2，
GD上的点ASF值均为（f1-z1）/w1
GK上的点ASF值均为（f2-z2）/w2
而(f1-z1)/w1=(f2-z2)/w2的！

所以GK和GD是ASF等高线~
所以 $\mathrm{ASF}(j, \lambda^i)$ 代表的意思是求j在 λ^i 上的“投影”。

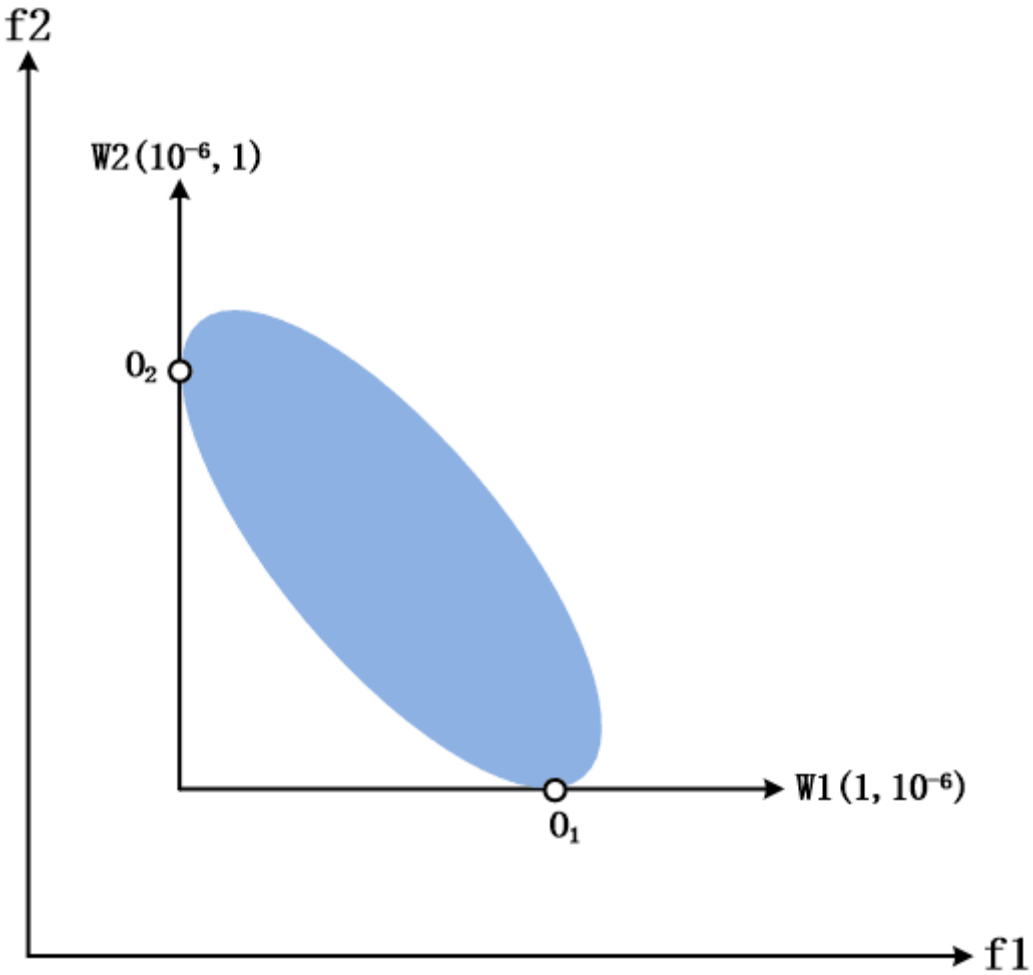
$$j^* = \arg\left(\min_{j=1:N} \left(\mathrm{ASF}\left(j, \lambda^i\right) \right) \right).$$

而minASF(J)是找出一个个体j，这个j在权重向量上的投影值是最小的，即该 轴上离理想点最近的点j，所以点0是我们要找的。



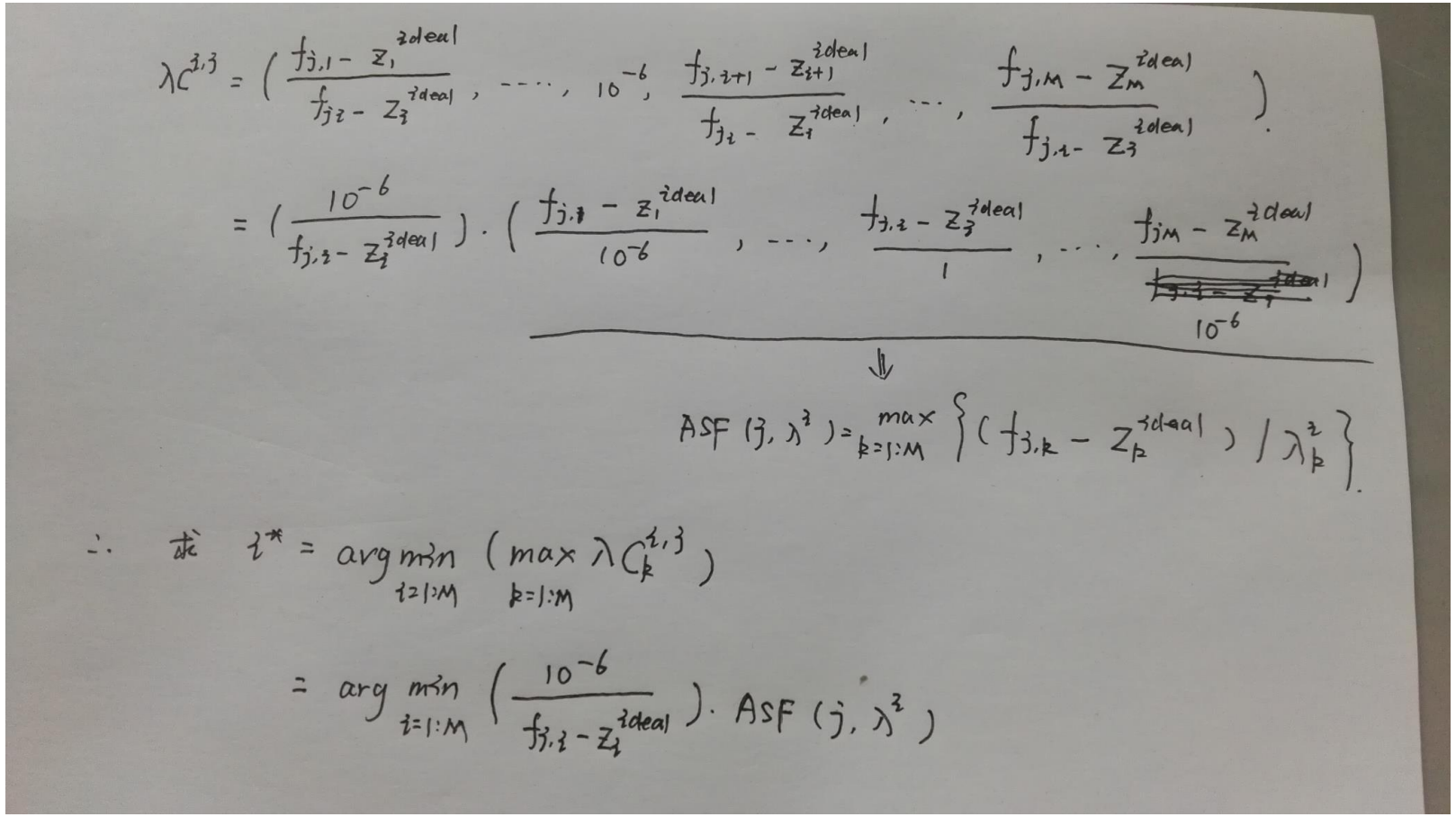
当权重向量变成一个坐标轴向量时，如w1,w2，我们得到就是整个种群的边界点O1，O2

$$\lambda^i = \left(10^{-6}, \ldots, 1, 10^{-6}, \ldots, 10^{-6}\right)^T \qquad (2)$$



我们引入分类向量 $\lambda c^{i,j}$（classification vector）将整个种群分成M个子种群（subpopulation）。公式4,5的目的是求出个体j最接近的目标轴i,就把j分到第i类。

$$\lambda c^{i,j} = \left(\lambda c_1^{i,j}, \ldots, \lambda c_i^{i,j}, \lambda c_{i+1}^{i,j}, \ldots, \lambda c_M^{i,j}\right)^T$$
$$= \left(\frac{f_{j,1} - z_1^{\text{ideal}}}{f_{j,i} - z_i^{\text{ideal}}}, \ldots, 10^{-6}, \frac{f_{j,i+1} - z_{i+1}^{\text{ideal}}}{f_{j,i} - z_i^{\text{ideal}}}, \ldots, \frac{f_{j,M} - z_M^{\text{ideal}}}{f_{j,i} - z_i^{\text{ideal}}}\right)^T. \quad (4)$$

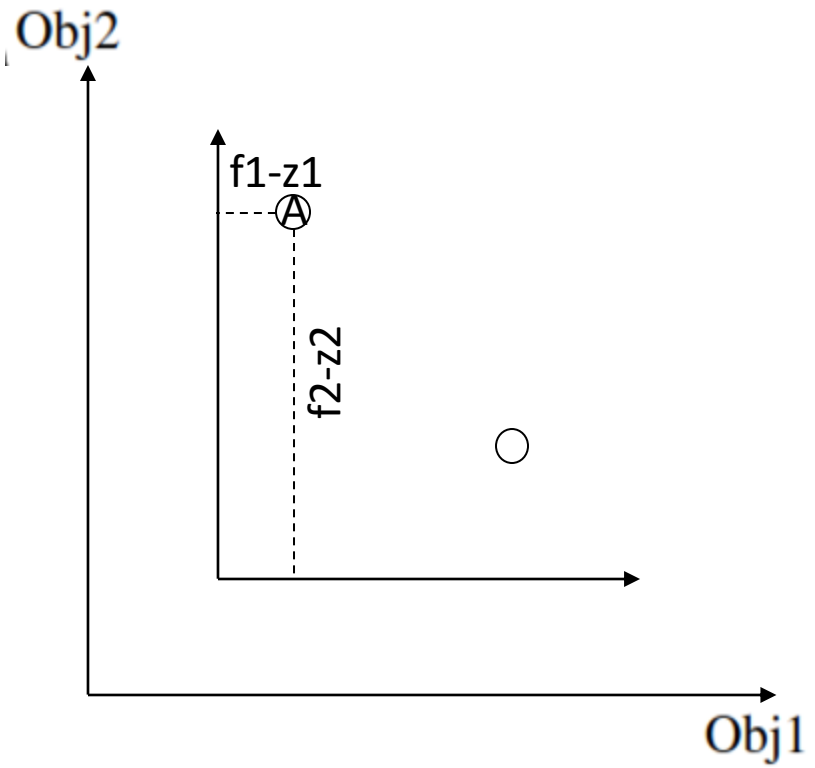


$$i^* = \arg\min_{i=1:M}\left(\max_{k=1:M} \lambda C_k^{i,j}\right). \quad (5)$$

$$\arg\min \left\{ \frac{f_{j2} - z_2^{ideal}}{f_{j1} - z_1^{ideal}}, \frac{f_{j1} - z_1^{ideal}}{f_{j2} - z_2^{ideal}} \right\}$$

$$= \arg\min \left\{ \frac{(f_{j2} - z_2^{ideal})^2}{\Delta}, \frac{(f_{j1} - z_1^{ideal})^2}{\Delta} \right\}$$

$j=1$ $\quad$ $j=2$.



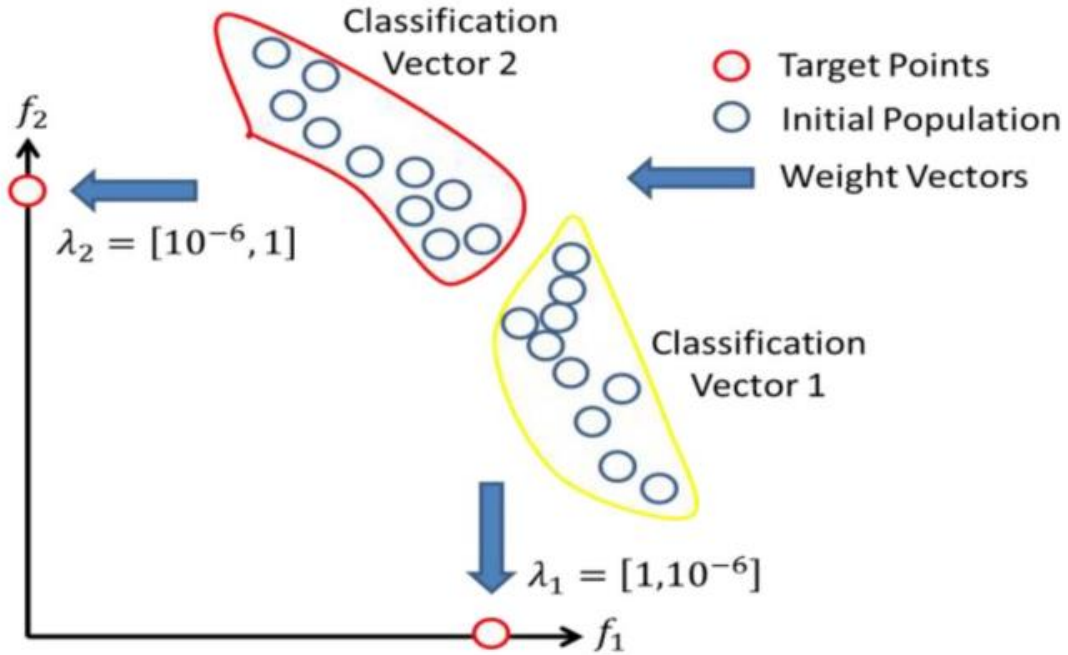


Fig. 2. Example of finding the target points.

Input: population size (N), the number of objectives (M), and the number of generations (num)

Step 1: Initialization

1) Generate an initial population with N individuals
2) Construct ideal point $z^{ideal} = \left(z_1^{ideal}, \ldots, z_M^{ideal}\right)^T$
3) Construct M weight vectors $\{\lambda^1, \lambda^2, \ldots, \lambda^M\}$ for ASF
4) For each individual j, its *ith* classification vector is $\lambda C^{i,j} = \left(\lambda C_1^{i,j}, \ldots, \lambda C_i^{i,j}, \lambda C_{i+1}^{i,j}, \ldots, \lambda C_M^{i,j}\right)^T$

Step 2: Classification of solutions

1) For objective i, its main classification vector is λC^i.
2) For each individual j, find $i^* = \arg \min_{i=1:M} (\max_{k=1:M} \lambda C_k^{i,j})$
3) Then, individual j belongs to the i^*th subpopulation

Step 3: Evolution of solutions

1) Generate offspring: for each subpopulation with size $S = N/M$, each time randomly choose two individuals as parents and generate offspring by crossover and mutation
2) Select S individuals from $2S$ individuals (combination of old population and offspring): sort all individuals by ASF values (the individual with the smallest ASF value comes first) and place first S individuals into the next generation.

Step 4: Reclassification using classification vectors. If the number of generations is larger than or equal to num, go to Step 5; otherwise, go back to Step 3.

Step 5: Identify Target Points (TPs): individual j^* is the TP of objective i,
such that $j^* = \arg(\min_{j=1:N}\{\max_{k=1:M}\{(f_{j,k} - z_k^{ideal})/\lambda_k^i\}\})$

Output: $TP = \{TP^1, TP^2, \ldots, TP^M\}$

Stage 2 :Diversity improvement

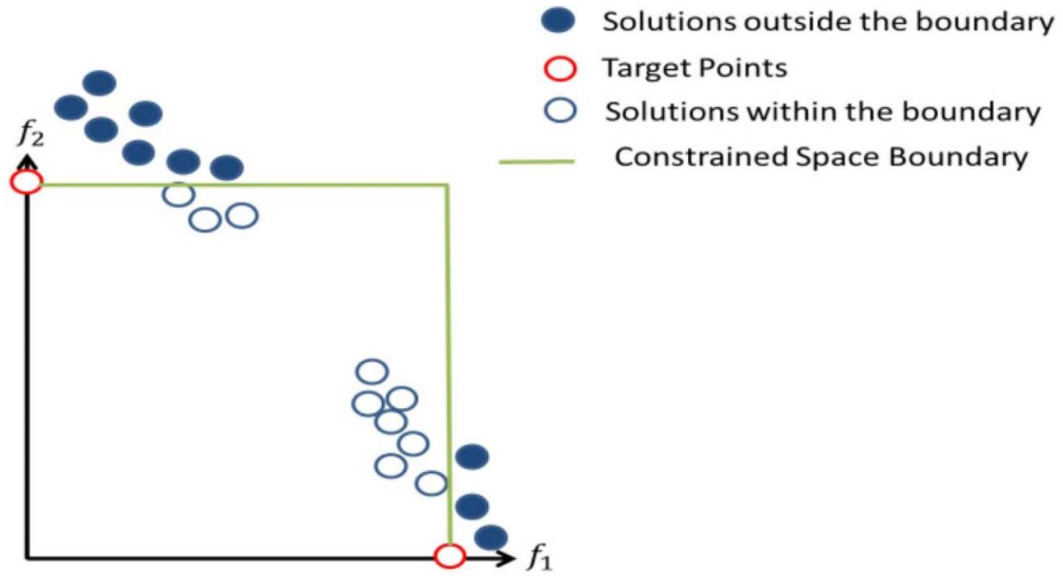
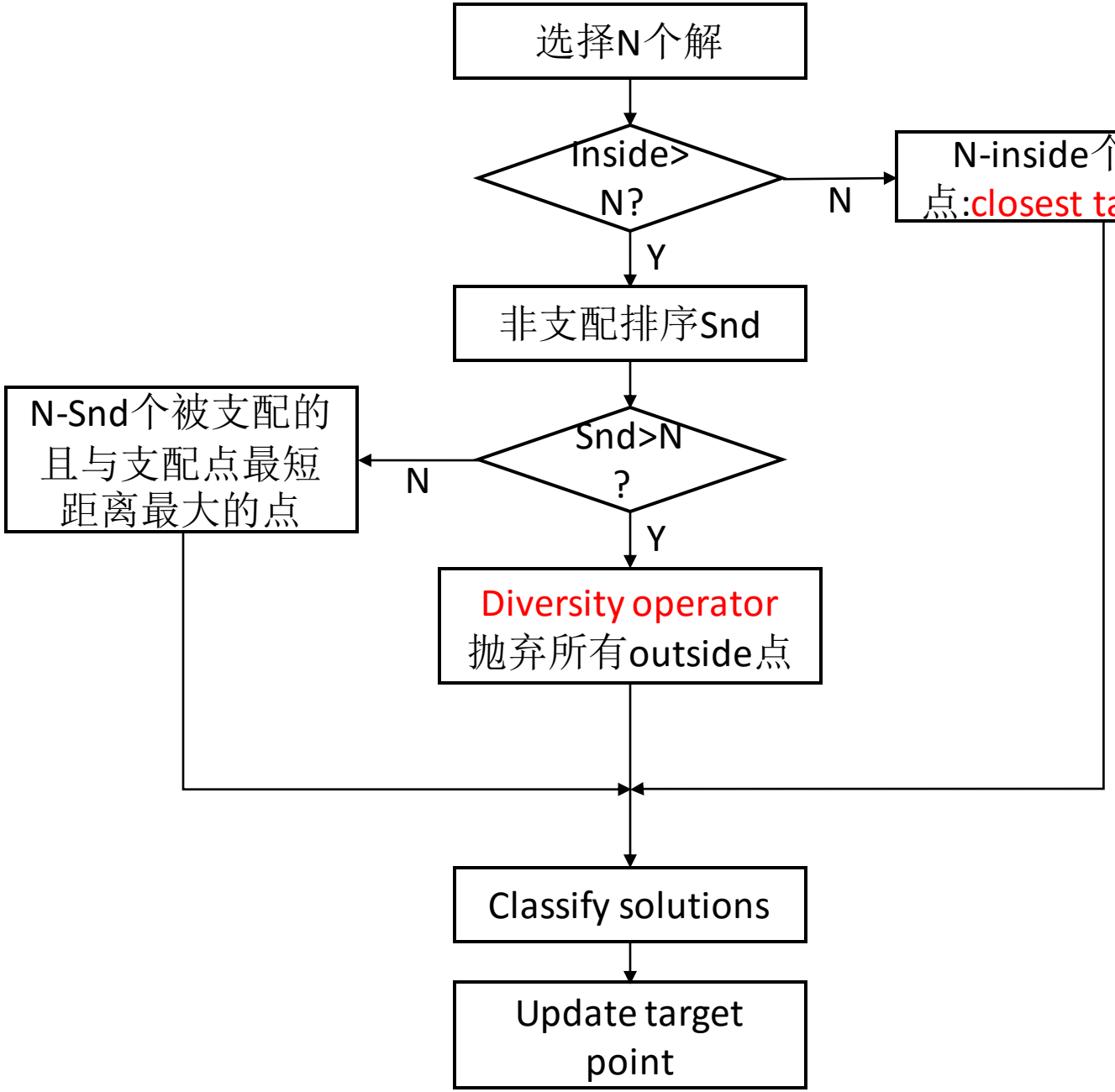


Fig. 4. Reduced objective space boundary.

Inside point:当一个个体在所有维度上的函数值均小于等于对应的target point值时

在边界内的点(inside)是对收敛性起作用的
在边界外的点(outside)只能使收敛性变差，所以只考虑提高多样性

所以，选择过程的基本思想是:尽量选择Inside点，多删少补



Stage 2 :Diversity improvement

Step 1: Initialization

1) Generate M subpopulations with size N/M around each TP:

2) Determine objective space boundary $(b_1, b_2, \dots, b_M)$: $b_i = \max(TP_i^1, TP_i^2, \dots, TP_i^M)$

If $\forall\, i \in \{1,2 \dots, M\}$, $f_i^j \leq b_i$; then Individual j is within objective space boundary

If $\exists\, i \in \{1,2 \dots, M\}$, $f_i^j > b_i$; then Individual j is outside of objective space boundary

Step 2: Generate offspring

1) Each time randomly choose two individuals among N solutions as parents and generate offspring using crossover and mutation

2) Classification:

Classify all individuals (combination of old population and offspring) into two groups:

The first group contains individuals within the objective space boundary

The second group contains individuals outside the objective space boundary

Step 3: Selection:

Selection of outside points

目的：inside点不足N时，需要一种策略将outside点补充
思想：target point 和middle point可以大概表示真实的pareto area,选择最接近这两种点的outside点。

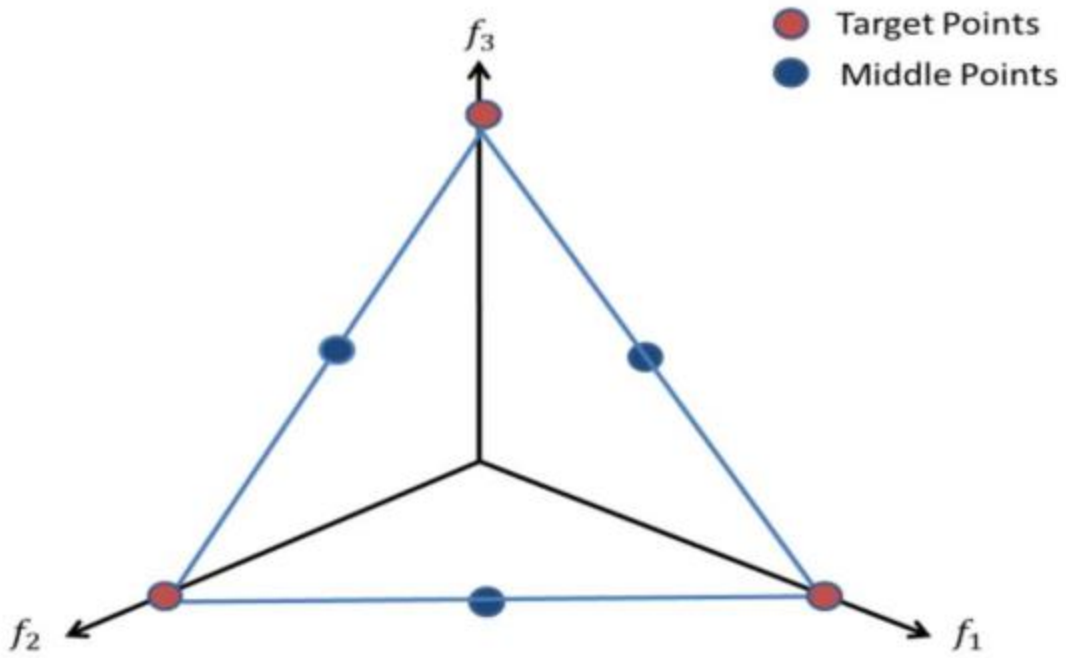
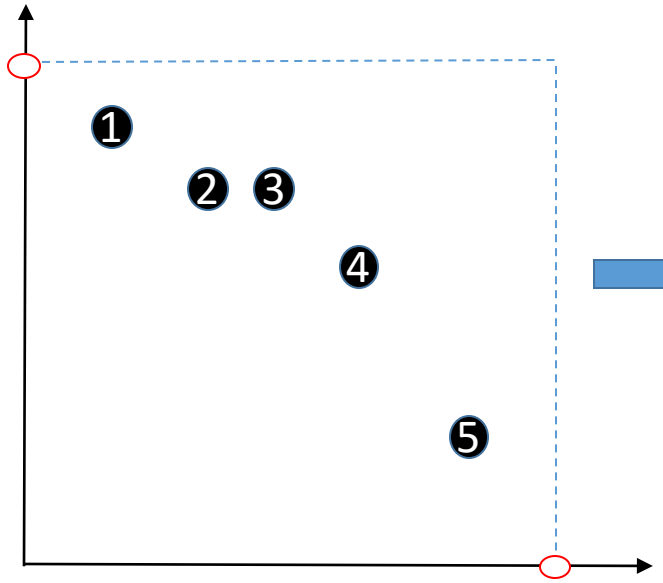


Fig. 5. Example of target points and their middle points.

Diversity operator

目的：消除非支配排序后，Snd>N的问题
思想：非支配关系下，为保持分布性去除靠近的点



A	1	2	3	4	5
1	-1	0.2	0.3	0.5	0.8
2		-1	0.1	0.3	0.6
3			-1	0.2	0.5
4				-1	0.3
5					-1

S

1	2	0.2
2	3	0.1
3	2	0.1
4	3	0.2
5	4	0.3

消除2,3并判断snd>N?
还是大于则重新计算1,4,5的距离，直到<N为止

A	1	4	5
1	-1	0.5	0.8
4		-1	0.3
5			-1

实验结果:

和下表五种算法比较，比较DTLZ1~DTLZ7,WFG1~WFG9在5维和10维上的结果，S-metric 即HV

TABLE III
SUMMARY OF STATISTICAL TEST RESULTS

MaOEA -R&D	vs. MOEA/D	vs. HypE	vs. GrEA	vs. NSGA-III	vs. ε-MOEA
IGD	+: 29 =: 3 -: 0	+: 24 =: 7 -: 1	+: 21 =: 10 -: 1	+: 25 =: 7 -: 0	+: 24 =: 8 -: 0
S-metric	+: 28 =: 4 -: 0	+: 13 =: 15 -: 4	+: 23 =: 6 -: 3	+: 21 =: 8 -: 3	+: 23 =: 8 -: 1

What can we do with a hyperplane

1、we can assign reference points on it——NSGA III

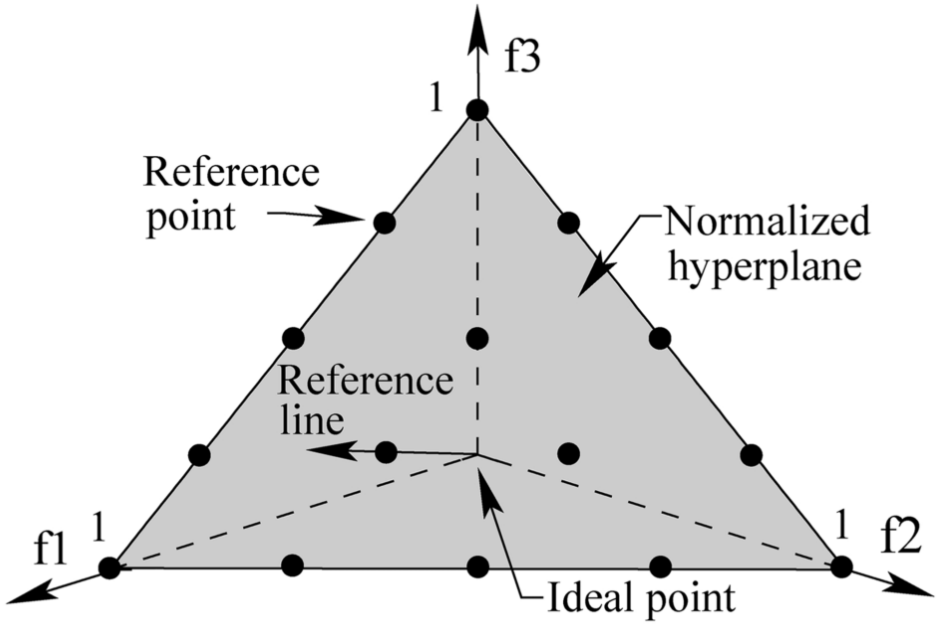
2、we can calculate distance from it——Knea

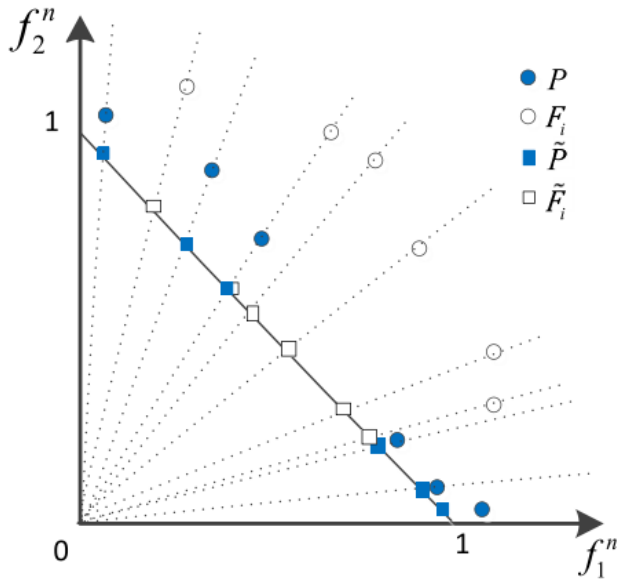
3、we can project point onto it——MaOEA-DDFC

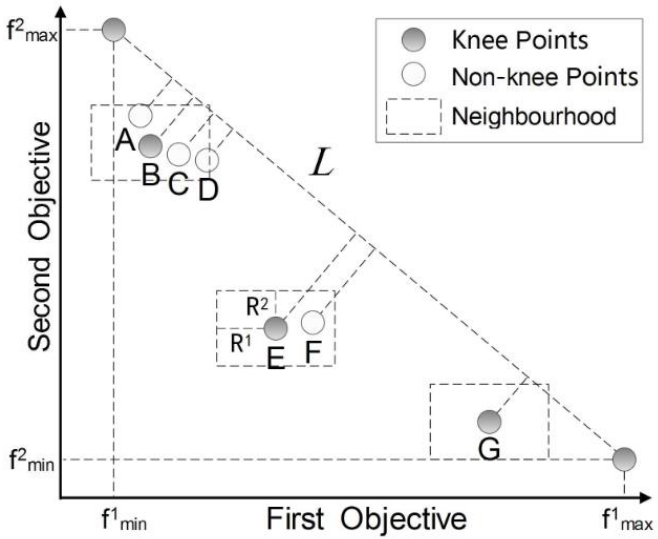
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超平面的选择可以决定进化算法的效果——如何确定超平面？

1、ASF

$$\mathrm{ASF}(j, \lambda^i) = \max_{k=1:M} \left\{ \left(f_{j,k} - z_k^{\mathrm{ideal}} \right) \Big/ \lambda_k^i \right\}$$

2、极值点直接构造

3、corner point构造

Future works

1、更合理的超平面构造方式

2、更动态的超平面指导作用

Thank you!